

5 MPEC

 Name

Learning objectives

- Define mathematical programming with equilibrium constraints (MPEC)
- Define bilevel programming and Stackelberg games
- Formulate and solve bilevel programming problems
- Propose problems that can be casted as MPECs

Bibliography

- Basic reading:
 - Bard, Jonathan F. Practical bilevel optimization: algorithms and applications. Vol. 30. Springer, 1998 (chapter 1).
 - Gabriel, S., Conejo. A., Fuller, D., Hobbs, B., and Ruiz, C. Complementarity modeling in energy markets. Springer, 2012 (chapter 3).
 - Z.Q. Luo, J.S. Pang and D. Ralph: Mathematical Programs with Equilibrium Constraints. Cambridge University Press, 1996.
- Further reading:
 - Hobbs, B. F., Metzler, C. B., Pang, J. S. (2000). Strategic gaming analysis for electric power systems: An MPEC approach. Power Systems, IEEE Transactions on, 15(2), 638-645.
 - Zugno, M., Morales, J. M., Pinson, P., Madsen, H. (2013). A bilevel model for electricity retailers' participation in a demand response market environment. Energy Economics, 36, 182-197.
 - Ruiz, C., Conejo, A. J. (2009). Pool strategy of a producer with endogenous formation of locational marginal prices. Power Systems, IEEE Transactions on, 24(4), 1855-1866.
 - Morales, J. M., Pinson, P., Madsen, H. (2012). A transmission-cost-based model to estimate the amount of market-integrable wind resources. Power Systems, IEEE Transactions on, 27(2), 1060-1069.
 - Pineda, S., Morales, J. M. (2014). Modeling the Impact of Imbalance Costs on Generating Expansion of Stochastic Units. arXiv preprint arXiv:1402.4593.

Example 5-1: Sequential game for power producers

In Example 4-7, we assumed that both producers had to determine their production simultaneously. Let us assume now that producer g_1 determines its production first, and afterwards producer g_2 does it. Both producers still behave as Nash-Cournot (strategic in quantities). Formulate an optimization problem to find the optimal production of g_1 and g_2 .

Solving Example 5-1 with GAMS (Example5-1.gms)

Solve the problem formulated in Example 5-1 using GAMS. Is the solution different from the one corresponding to Example 4-8? Why? *Tip: use `nlp` in GAMS.*



Mathematical Programming with Equilibrium Constraints



Bilevel programming / hierarchical optimization / Stackelberg game

✚ Example 5-2: Capacity expansion of conventional generating unit

In previous examples $\bar{x}_g$ was a given parameter representing the maximum output of the generating units. In this example, an optimization problem that determines the optimal capacity $\bar{x}_g$ in order to maximize the profit of a power producer has to be formulated. The production cost function of the unit to be built is denoted by $\mathcal{C}_g(x_g) = c_g x_g$, while that corresponding to each existing generating unit is represented by $\mathcal{C}_{\tilde{g}}(x_{\tilde{g}}) = c_{\tilde{g}} x_{\tilde{g}}$. The maximum capacity of existing units is denoted by $\bar{x}_{\tilde{g}}$. $\bar{X}_g$ represents the maximum capacity to be installed, while the investment cost function is indicated by $\mathcal{C}_g^I(\bar{x}_g) = i_g \bar{x}_g$. The price of electricity is determined by solving a market clearing that maximizes the social welfare to satisfy a fixed demand d (perfect competition). This demand is not constant throughout the planning horizon, and its variation is approximated using N^W steps so that during t_w hours of the planning horizon the demand level is equal to d_w . Propose a formulation and justify why it is appropriate for this problem.

📌 Example 5-3: Capacity expansion of conventional generating unit (II)

Replace the lower-level problem of Example 5-2 by its KKT conditions, provided they are necessary and sufficient for optimality, and formulate the problem as a mixed-linear optimization problem. *Tip: Assume that the total capacity of the new generating unit can be discretized in N^B equal blocks as follows: $\bar{x}_g = \sum_b v_b \hat{x}_g$, where v_b is a binary variable and $\hat{x}_g$ is the block size. Note that $\bar{X}_g = N^B \hat{x}_g$. Tip2: use the Fortuny-Amat linearization for complementarity constraints of next page. Tip3: use the linearization of the product of binary and continuous variable of next page.*



Fortuny-Amat linearization for complementarity constraints



Linearization of the product of binary and continuous variable

Solving example 5-3 (Example5-3.gms)

Use GAMS to solve the problem formulated in example 5-3. Use the data below. g_2 and g_3 are existing units, and the optimal capacity of g_1 has to be determined. Assume that the investment cost are $i_{g_1} = \$50000/\text{MWyear}$, the maximum capacity $\bar{X}_{g_1} = 1000\text{MW}$ and the block size $\hat{x}_{g_1} = 50\text{MW}$.

Unit	$\underline{x}_g$	$\bar{x}_g$	c_g	Demand steps	t_w	d_w
g_1	0	?	5	w_1	2920	325
g_2	0	400	10	w_2	2920	575
g_3	0	400	15	w_3	2920	750

What happens if the value of the large constant M is set to a very low value?

What happens if the value of the large constant M is set to a very high value?

Can you think about a way to reduce numerical issues of this problem?

📌 Example 5-4: Capacity expansion of conventional generating unit (III)

Replace the lower-level problem of Example 5-2 by its primal-dual formulation, provided these conditions are necessary and sufficient for optimality, and formulate the problem as a mixed-linear optimization problem. *Tip: KKT conditions still hold. Tip2: Assume that the total capacity of the new generating unit can be discretized in N^B equal blocks as follows: $\bar{x}_g = \sum_b v_b \hat{x}_g$, where v_b is a binary variable and $\hat{x}_g$ is the block size. Note that $\bar{X}_g = N^B \hat{x}_g$.*

Solving example 5-4 (Example5-4.gms)

Use GAMS to solve the problem formulated in example 5-4. Use the data provided in example 5-3. Do you obtain the same solution?

Scalability of capacity expansion problem

Use the codes Example5-3.gms and Example5-4.gms and solve them using 500 steps for the demand curve. Generate the demand values using the GAMS function “normal” with an average demand of 575 MW and a standard deviation of 200 MW. Use the same duration for all time steps. Provide the computational time for each model and compare them.



Discuss the advantages and disadvantages of formulations of examples 5-3 and 5-4



Discuss further research about formulations of examples 5-3 and 5-4

✦ Example 5-5: Capacity expansion of conventional generating unit (IV)

The main purpose of this exercise is to include the network in the capacity expansion problem. The parameter $h_{\tilde{g}n}$ is a unit-bus indicator equal to 1 if existing unit $\tilde{g}$ is located at bus n , and 0 otherwise. The demand at each node is d_{nw} during t_w hours, the flow through line l is denoted by f_l (with a maximum value of $\bar{f}_l$), and i_{ln} is an line-bus indicator parameter equal to 1/-1 for the sending/receiving bus of line l and to 0 otherwise. Formulate a bilevel optimization problem to determine the optimal capacity to be allocated at each bus $\bar{x}_{gn}$ of generating units with constant marginal cost c_g and an investment cost determined by $\sum_n i_g \bar{x}_{gn}$. The maximum capacity to be installed at all buses cannot be higher than $\bar{X}_g$. The capacity installed at each bus is discretized in blocks of size $\hat{x}_g$. The existing units also have a linear marginal cost of $c_{\tilde{g}}$ and a capacity of $\bar{x}_{\tilde{g}}$.

📌 Example 5-6: Capacity expansion of conventional generating unit (V)

Provide a mixed-linear equivalent formulation of the problem described in Example 5-5.

□ Impact of network in capacity expansion (Example5-6.gms)

Use GAMS to solve the formulation corresponding to example 5-6. Use the following data: $c_g = 8\$/\text{MWh}$, $\bar{X}_g = 500\text{MW}$, $i_g = \$50000/\text{MWyear}$, $\hat{x}_g = 50\text{MW}$. Determine the installed capacities at each bus of the system.

Unit	Bus	$\underline{x}_g$	$\bar{x}_g$	c_g	Bus	d_{nw}	t_w	Line	B_l	$\bar{f}_l$
g_1	n_1	0	250	5	w_1	300	2920	$l_1(n_1 - n_2)$	100	50
g_2	n_2	0	250	10	w_2	400	2920	$l_2(n_1 - n_3)$	100	100
g_3	n_3	0	250	16	w_3	500	2920	$l_3(n_2 - n_3)$	100	100

Do optimal capacity expansion decisions change if the network is disregarded, i.e., with $\bar{f}_l = 1000$?



Discuss further applications of MPEC and formulate one of them